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COMPUTATIONAL THINKING AS AN EPISTEMOLOGICAL CATALYST: A THEORETICAL INTEGRATION BETWEEN MATHEMATICS EDUCATION AND TRANSDISCIPLINARITY

PENSAMENTO COMPUTACIONAL COMO CATALISADOR EPISTEMOLÓGICO: UMA ARTICULAÇÃO TEÓRICA ENTRE A EDUCAÇÃO MATEMÁTICA E A TRANSDISCIPLINARIDADE

PENSAMIENTO COMPUTACIONAL COMO CATALIZADOR EPISTEMOLÓGICO: UNA ARTICULACIÓN TEÓRICA ENTRE LA EDUCACIÓN MATEMÁTICA Y LA TRANSDISCIPLINARIEDAD

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ABSTRACT | Mathematics education continues to struggle with procedural fragmentation, decontextualized instruction, and an overreliance on mechanical reproduction. This article proposes Computational Thinking (CT) not as an isolated technical skill, but as an epistemological catalyst capable of transforming mathematical learning and fostering transdisciplinary practices. Grounded in the Theory of Objectification, the study theoretically integrates CT, transdisciplinarity, and semiotic mediation to reposition mathematics as a dynamic, culturally situated practice. Through conceptual synthesis and bibliographic analysis, we demonstrate how CT reconfigures learning through three interconnected mechanisms: (1) the semiotic externalization of mathematical reasoning via algorithms and computational models; (2) the cultivation of an investigative classroom ethos that normalizes iteration, debugging, and collaborative sense-making; and (3) the organic emergence of transdisciplinary inquiry by bridging mathematical formalization with complex real-world problems. The article further outlines practical implications for curriculum design, teacher preparation, process-oriented assessment, and educational equity, while cautioning against the reduction of CT to mere programming instruction. We conclude by proposing a methodological agenda for future empirical research, emphasizing longitudinal, cross-cultural, and design-based studies to validate and operationalize this theoretical framework. Ultimately, this work advocates for a paradigm shift in mathematics education, where CT functions as a transformative mediator that aligns disciplinary learning with the cognitive and ethical demands of 21st-century citizenship.

KEYWORDS: Problem solving, Critical thinking, Epistemology, Semiotic mediation, Investigative ethos.

RESUMO | A educação matemática enfrenta desafios persistentes relacionados à fragmentação curricular, à descontextualização dos conteúdos e à ênfase na reprodução mecânica de procedimentos. Este artigo propõe o Pensamento Computacional (PC) não como uma ferramenta técnica isolada, mas como um catalisador epistemológico capaz de transformar a aprendizagem matemática e fomentar práticas transdisciplinares. Ancorado na Teoria da Objetivação, o estudo integra teoricamente o PC, a transdisciplinaridade e a mediação semiótica para reposicionar a matemática como uma prática dinâmica e culturalmente situada. Por meio de síntese conceitual e pesquisa bibliográfica, demonstra-se como o PC reconfigura a aprendizagem por meio de três mecanismos interligados: (1) a externalização semiótica do raciocínio matemático via algoritmos e modelos computacionais; (2) o cultivo de um *ethos investigativo* na sala de aula, que normaliza a iteração, a depuração e a construção colaborativa de sentido; e (3) a emergência orgânica de investigações transdisciplinares, ao conectar a formalização matemática a problemas complexos do mundo real. O artigo detalha ainda implicações práticas para o desenho curricular, a formação de professores, a avaliação processual e a equidade educacional, alertando contra a redução do PC ao mero ensino de programação. Conclui-se propondo uma agenda de pesquisa empírica futura, com ênfase em estudos longitudinais, comparativos e baseados em design, para validar e operacionalizar este referencial teórico. Em síntese, defende-se uma mudança de paradigma na educação matemática, na qual o PC atua como mediador transformador, alinhando a aprendizagem disciplinar às demandas cognitivas e éticas da cidadania contemporânea.

PALAVRAS-CHAVE: Resolução de problemas, Pensamento crítico, Epistemologia, Mediação semiótica, Ethos investigativo.

RESUMEN | La educación matemática enfrenta desafíos persistentes relacionados con la fragmentación curricular, la descontextualización de los contenidos y un enfoque excesivo en la reproducción mecánica de procedimientos. Este artículo propone el Pensamiento Computacional (PC) no como una herramienta técnica aislada, sino como un catalizador epistemológico capaz de transformar el aprendizaje matemático y fomentar prácticas transdisciplinarias. Fundamentado en la Teoría de la Objetivación, el estudio integra teóricamente el PC, la transdisciplinariedad y la mediación semiótica para reposicionar las matemáticas como una práctica dinámica y culturalmente situada. Mediante síntesis conceptual e investigación bibliográfica, se demuestra cómo el PC reconfigura el aprendizaje a través de tres mecanismos interconectados: (1) la externalización semiótica del razonamiento matemático mediante algoritmos y modelos computacionales; (2) el cultivo de un *ethos* investigativo en el aula que normaliza la iteración, la depuración y la construcción colaborativa de significados; y (3) la emergencia orgánica de indagaciones transdisciplinarias al vincular la formalización matemática con problemas complejos del mundo real. El artículo esboza además implicaciones prácticas para el diseño curricular, la formación docente, la evaluación procesual y la equidad educativa, advirtiendo contra la reducción del PC a la mera enseñanza de programación. Se concluye proponiendo una agenda de investigación empírica futura que priorice estudios longitudinales, comparativos y basados en *diseño* para validar y operacionalizar este marco teórico. En definitiva, se aboga por un cambio de paradigma en la educación matemática, donde el PC funcione como un mediador transformador que alinee el aprendizaje disciplinar con las demandas cognitivas y éticas de la ciudadanía del siglo XXI.

PALABRAS CLAVE: Resolución de problemas, Pensamiento crítico, Epistemología, Mediación semiótica, Ethos investigativo.

1. INTRODUCTION

In recent years, mathematics education faces persistent challenges manifested in curricular fragmentation, the decontextualization of content, and students' consequent resistance toward the subject. As noted by several authors (Budhatoki et al., 2025; Alrashidi & Alreshidi, 2026; Sujadi et al., 2025; Yang et al., 2025; Altarawneh, 2025), teaching practices in many countries remain centered on the reproduction of procedures. This persistence undermines the construction of meaning and reduces mathematics to a collection of techniques disconnected from real-world problems or other fields of knowledge. This scenario highlights the urgency of rethinking teaching models, which largely remain bound to isolated disciplinary approaches and fail to engage with the complexity of the contemporary world.

In this context, CT emerges as a fruitful alternative. Initially introduced by Wing (2006, 2008) as a fundamental mode of thought rather than a synonym for programming, CT emphasizes cognitive processes such as decomposition, pattern recognition, abstraction, and algorithmic design. The research landscape surrounding CT has evolved significantly, expanding from computer science into mathematics education as a fertile ground for 21st-century literacy (Yadav et al., 2016; Küçükkaydın et al., 2024; Pinheiro & Santos, 2025). Recent studies have begun to explore not only students' acquisition of problem-solving skills but also the broader epistemological and cognitive dimensions of this integration. Scholars have increasingly framed CT within the notion of computational literacies, highlighting how learners' engagement with digital tools mediates meaning-making and fosters more inclusive, critical, and reflective forms of disciplinary participation (Kafai & Proctor, 2022). Furthermore, emerging evidence suggests that CT-based pedagogies can significantly enhance cross-domain competencies and foster integrative thinking across STEM fields (Kong & Abelson, 2019; Buitrago-Flórez et al., 2021).

Despite these promising directions, empirical and theoretical discussions often treat CT as a technical scaffold or a standalone cognitive skill, overlooking its deeper role in shaping how mathematical knowledge is culturally and semiotically constituted. Most existing studies focus on performance metrics or procedural fluency, leaving the underlying processes of meaning-making and conceptual objectification underexamined. To address this gap, a theoretical lens is needed that moves beyond instrumental outcomes to explain how computational practices actively transform students' engagement with mathematical ideas. This necessity calls for an integrative framework that connects CT with established cultural-historical perspectives and transdisciplinary horizons, thereby repositioning mathematics as a dynamic, socially mediated practice.

Grounded in the Theory of Objectification (TO), this article advances precisely such a framework. The TO posits that mathematical knowledge is not individually internalized but gradually brought into conscious awareness through semiotic mediation – where signs, artifacts, and social interaction shape collective understanding (Radford, 2008, 2013, 2014, 2016). Within this perspective, CT functions not as a mere technical add-on, but as a rich semiotic infrastructure. Algorithms, computational models, and dynamic visualizations serve as cultural artifacts that externalize abstract reasoning, making it visible, negotiable, and open to refinement. Moreover, the iterative nature of computational work – characterized by debugging, testing, and collaborative revision – cultivates an investigative ethos that normalizes error, encourages intellectual risk-taking, and sustains dialogic engagement. When analyzed through the TO, these

CT-mediated practices reveal how learners co-construct mathematical meaning while developing the epistemic flexibility required to navigate complex, real-world problems.

The relevance of this discussion lies in recognizing that 21st-century citizens require competencies to confront challenges that extend beyond any single domain of knowledge. Mathematics, when connected to CT and examined through the TO, can be understood as a cultural practice that contributes to the formation of critical, creative individuals capable of traversing diverse knowledge domains. In this sense, this work aligns with the growing movement to value transdisciplinary approaches that acknowledge complexity as an essential epistemological horizon for contemporary education.

Thus, this article contributes by examining the epistemological role of CT in the constitution of mathematical knowledge, proposing an integrated theoretical model that articulates semiotic mediation, an investigative ethos, and transdisciplinarity. Rather than offering empirical findings, it synthesizes cultural-historical semiotics, transdisciplinary theory, and contemporary CT frameworks to reposition mathematics education. The argument is intended to stimulate scholarly dialogue and inform curriculum design, teacher education, and future empirical research.

2. ANCHORING COMPUTATIONAL THINKING IN SEMIOTICS AND TRANSDISCIPLINARITY

The articulation developed in this article rests on a network of four interrelated dimensions: the reconceptualization of mathematics education, the epistemological nature of computational thinking, transdisciplinarity as an integrative horizon, and the TO as the analytical lens that binds them. Rather than treating these domains in isolation, this section synthesizes them into a coherent foundation for understanding CT as a transformative force in mathematical learning.

2.1 Mathematics education beyond algorithmia

Traditional mathematics education has frequently been constrained by an emphasis on procedural fluency, algorithmic reproduction, and symbolic manipulation. This orientation risks reducing the discipline to a closed system of rules, detached from meaning, creativity, and authentic problem-solving. As the National Research Council (2001) emphasizes, mathematical proficiency requires conceptual understanding, adaptive reasoning, and the capacity to model real-world phenomena – competencies that remain underdeveloped when instruction prioritizes mechanical execution. Moving beyond what may be termed *algorithmia* necessitates reframing mathematics as a cultural and social practice. Drawing on socio-cultural perspectives, Radford (2008) posits that mathematical knowledge is not merely internalized individually but co constructed through socially mediated activities involving language, artifacts, and signs. In this view, mathematics education must cultivate reasoning, conjecture, argumentation, and critical inquiry. Such a reconceptualization creates fertile ground for integrating computational thinking, as both domains share an orientation toward abstraction, modeling, and the dynamic construction of knowledge.

2.2 Computational Thinking as an epistemic practice

Computational Thinking, initially framed by Wing (2006,2008) as a fundamental mode of thought comparable to literacy, transcends the narrow association with programming. It encompasses cognitive processes such as decomposition, pattern recognition, abstraction, and algorithmic design – strategies that structure problem-solving across diverse contexts. Subsequent frameworks, such as Brennan & Resnick's (2012) tripartite model, expand CT to include conceptual knowledge, iterative practices (e.g., debugging, remixing), and perspectival shifts toward creativity and collaboration. When integrated into mathematics, CT functions not as a technical add-on but as a semiotic infrastructure that externalizes reasoning. Algorithms, models, and computational representations serve as cultural artifacts that make abstract mathematical ideas visible, negotiable, and open to refinement. This shift from technical training to epistemic practice positions CT as a meta-cognitive resource that amplifies students' capacity to engage with mathematics as both a formal system and a tool for interpreting reality.

2.3 Transdisciplinarity as an epistemological horizon

Contemporary societal challenges are inherently complex and cannot be adequately addressed within disciplinary silos. While multidisciplinarity juxtaposes fields and interdisciplinarity fosters limited dialogue, transdisciplinarity seeks to unify knowledge through shared ontological and epistemological commitments (Nicolescu, 2002). It recognizes that authentic problem-solving requires the integration of mathematical, scientific, technological, and humanistic perspectives into coherent frameworks of understanding (Galati, 2023; Sue, 2024). Within mathematics education, transdisciplinarity challenges curricular compartmentalization and demands pedagogical approaches that mirror the interconnectedness of the real world. CT plays a pivotal role in this horizon by functioning as a mediating language that enables cross-domain dialogue. By providing cognitive and methodological tools to simulate, test, and refine complex systems, CT transforms transdisciplinarity from an abstract ideal into an operational necessity. Adopting this horizon requires cultivating dispositions of openness, critical reflection, and epistemic flexibility, positioning mathematics as a bridge rather than a barrier between knowledge domains.

2.4 The Theory of Objectification as the integrative lens

The TO (Radford, 2008, 2013, 2014, 2016) offers a cultural-historical framework for understanding howmathematical knowledge emerges in classroom settings. Radford (2013, 2014) conceptualizes learning not as the internalization of pre-existing information, but as objectification – the process through which learners, mediated by signs, artifacts, and social interaction, gradually bring mathematical concepts into conscious awareness and shared understanding. Central to this process is semiotic mediation: tools such as algebraic notation, diagrams, or computational models do not merely transmit knowledge; they actively shape how meaning is constructed and negotiated. Equally important is the notion of ethos, which refers to the collective norms, values, and dispositions that structure the learning environment (Amundrud et al., 2022; Ting & Wang, 2026). A classroom ethos that values exploration, tolerates error, and encourages collaboration creates optimal conditions for objectification to occur. Viewed through this lens, CT is neither a purely technical skill nor an isolated cognitive competence. Rather, it is a socio-cultural practice that enriches the semiotic landscape of mathematics education, aligns with the iterative and collaborative nature of knowledge construction, and provides the structural conditions for students to objectify complex mathematical ideas (Radford, 2016). This theoretical integration establishes the foundation for examining how CT operates as an epistemological catalyst, a theme developed in Section 4.

2.5 Operational definitions of core constructs

To ensure conceptual clarity before examining pedagogical implications, this article adopts the following operational descriptions of its three foundational constructs:

i. *Semiotic mediation* refers to the process by which cultural artifacts (e.g., algorithms, dynamic visualizations, formal notations) structure cognitive activity and enable the externalization, negotiation, and iterative refinement of mathematical reasoning. Operationally, it functions as a representational mechanism that transforms internal cognition into shared, observable, and modifiable artifacts.
ii. *Investigative ethos* denotes the collectively established classroom norms that systematically treat error, iteration, and collaborative debugging as constitutive of knowledge construction rather than as deviations from procedural correctness. Operationally, it manifests through pedagogical routines that prioritize hypothesis testing, version control, and dialogic critique over compliance with predetermined solution paths.
iii. *Transdisciplinarity* is understood as an epistemological orientation that dissolves rigid disciplinary boundaries by requiring the simultaneous mobilization of multiple knowledge systems to address complex, ill-structured problems. Operationally, it emerges when learning tasks demand the integration of mathematical formalization, scientific modeling, and contextual reasoning within a unified problem-solving framework.

These definitions delineate the conceptual boundaries of the constructs that will be examined in the following sections. Their pedagogical implications, structural reconfigurations, and relationships with computational thinking are addressed separately in Sections 4 and 5.

3. METHODOLOGICAL APPROACH

This study adopts a qualitative, theoretical-analytical approach grounded in a bibliographic research. Rather than generating empirical data through fieldwork or experimentation, the research proceeds through conceptual elaboration and critical synthesis of theoretical frameworks. This methodological choice is justified by the nature of the research problem: the aim is not to measure or observe specific educational outcomes, but to construct an integrated theoretical understanding of how CT can function as an epistemological catalyst in mathematics education, particularly when examined through the lens of the TO.

The bibliographic corpus was assembled through searches conducted in Scopus database, using the descriptors “computational thinking,” “mathematics education,” “Theory of Objectification,” “semiotic mediation,” “transdisciplinarity,” and “epistemology in education.” Priority was given to open-access peer-reviewed articles published in the last two decades (2006-2026), and foundational theoretical works, with particular attention to seminal contributions Wing (2006), Radford (2013), and Nicolescu (2002), that anchor the conceptual framework of the study. The selection criteria emphasized theoretical relevance, methodological rigor, and direct engagement with the intersection of CT and mathematics education.

The analytical procedure followed three interrelated steps. First, a conceptual mapping was conducted to identify and delineate the core theoretical constructs at stake: CT as a form of thought, the TO's notions of objectification, semiotic mediation, and ethos, and the epistemological principles underlying transdisciplinarity. Second, the relationships among these constructs were examined through a process of theoretical triangulation, in which insights from multiple theoretical traditions – cultural-historical theory, semiotics, and philosophy of education – were brought into dialogue to illuminate the epistemological role of CT. Third, the articulations identified were interpreted in terms of their implications for educational practice, including curriculum design, teacher education, and assessment.

This theoretical-integrative approach is consistent with established practices in educational research, where the construction of conceptual frameworks constitutes a rigorous and productive form of scholarly inquiry (Jabareen, 2009). By drawing systematically on a diverse body of literature and subjecting it to critical interpretive analysis, the study aims to generate theoretically grounded insights that can orient both future empirical research and the practical transformation of mathematics education. The contribution of this methodology lies not in the production of statistical or experimental evidence, but in the elaboration of a coherent theoretical architecture that repositions CT as a meaningful epistemological and pedagogical resource.

4. HOW COMPUTATIONAL THINKING RECONFIGURES MATHEMATICAL LEARNING

Building on the theoretical integration established in Section 2, this section advances three structured propositions that articulate how CT operates as an epistemological catalyst in mathematics education. Rather than presenting observational or empirical claims, these propositions synthesize cultural-historical semiotics, transdisciplinary theory, and contemporary CT frameworks to theorize the mechanisms through which computational practices transform mathematical learning.

4.1 The semiotic mediation of Computational Thinking

CT introduces a rich set of semiotic resources that expand how mathematical concepts are represented, manipulated, and shared. Processes such as decomposition, abstraction, and algorithmic design function not merely as technical steps but as semiotic actions that structure complex ideas into executable forms. When students construct an algorithm or a computational model, they produce symbolic artifacts that encapsulate their reasoning, making it visible, negotiable, and open to refinement (Weintrop et al., 2016; Brennan & Resnick, 2012). From the perspective of objectification, these artifacts serve as cultural mediators that bridge internal cognitive processes and shared mathematical understanding (Radford, 2013). Unlike traditional mathematics instruction, which often relies heavily on static symbolic notation, CT enriches the semiotic landscape through dynamic visualizations, interactive simulations, and executable models. These representations enable learners to test hypotheses, observe systemic behaviors, and iteratively adjust parameters. For instance, modeling the spread of an epidemic requires students to translate biological assumptions into mathematical structures (e.g., differential equations or recursive algorithms) and then operationalize them computationally. The resulting simulation does not merely calculate outcomes; it externalizes the underlying mathematical relationships, allowing students to interrogate assumptions, identify contradictions, and collaboratively refine their models. In this way, CT shifts learning from the passive application of formulas to the active construction of meaning, positioning mathematical knowledge as an evolving artifact of social and semiotic negotiation.

4.2 Cultivating an investigative ethos

Beyond its representational capacity, CT fundamentally reshapes the normative and affective dimensions of the learning environment. The iterative practices inherent to computational work – debugging, versioning, testing, and remixing – establish a classroom culture where error is reframed as an essential component of inquiry rather than a marker of failure. This contrasts sharply with traditional mathematics settings, where mistakes are frequently stigmatized and compliance is prioritized over exploration (Khasawneh et al., 2023; Kyaruzi et al., 2020).

For instance, when ninth grade students program a Scratch simulation to model population growth under resource constraints, an unexpected population crash reveals a logical flaw in their birth-rate condition. Rather than signaling failure, this "bug" becomes a object of collective inquiry: students trace variable updates, debate alternative formulations, and revise the algorithm collaboratively. Through this debugging cycle, error ceases to be a personal deficit and becomes a semiotic resource that deepens mathematical objectification.

Within a CT-infused classroom, the learning ethos becomes one of systematic experimentation. Students are encouraged to formulate conjectures, implement them computationally, analyze discrepancies, and revise their approaches. This cycle cultivates resilience, metacognitive awareness, and intellectual risk-taking. Moreover, computational tasks are inherently collaborative: learners share code, critique algorithmic efficiency, and co-construct solutions. Such distributed problem-solving not only manages cognitive complexity but also fosters dialogic engagement, mutual accountability, and the collective negotiation of meaning. By normalizing iteration and collaborative sense-making, CT transforms the classroom into a community of practice where curiosity, persistence, and epistemic agency become the defining norms of mathematical engagement.

4.3 Transdisciplinarity as an intrinsic outcome

A further consequence of integrating CT into mathematics education is the natural emergence of transdisciplinary learning. Contemporary challenges are rarely confined to single disciplines, and their resolution demands the simultaneous mobilization of diverse knowledge systems. CT provides the methodological scaffolding that enables this integration, functioning as a cognitive bridge across domains (Nicolescu, 2002). Because computational modeling inherently requires the identification of variables, the formulation of relationships, and the simulation of dynamic systems, it organically compels learners to draw upon multiple fields of knowledge.

For instance, consider a classroom activity in which students develop a computational model to simulate deforestation dynamics in the Amazon. To construct the model, they must integrate mathematical reasoning (e.g., rates of change and proportionality), ecological knowledge (e.g., vegetation recovery and human impact), and algorithmic logic (e.g., conditional rules governing land use). As students iteratively refine their simulations and compare outcomes, the computational artifact functions as a semiotic mediator through which assumptions are externalized, contested, and collectively objectified, making transdisciplinary reasoning both necessary and observable.

Similar dynamics emerge in contexts such as urban traffic flows or ecological predator-prey systems. Addressing such problems requires students to integrate mathematical formalization, scientific principles, and computational simulation within a unified framework. The complexity of the task itself dictates that disciplinary boundaries be crossed, not as an external curricular mandate, but as an epistemic necessity. Through CT, learners develop cognitive flexibility and the capacity for abstraction that allows them to transfer reasoning strategies across contexts. Transdisciplinarity, therefore, ceases to be an imposed pedagogical ideal and instead emerges as a structural consequence of authentic, computationally mediated inquiry. This alignment between CT practices and transdisciplinary problem-solving prepares students to navigate interconnected knowledge landscapes with analytical rigor and adaptive competence.

5. TRANSLATING THEORY INTO EDUCATIONAL PRACTICE

The theoretical articulation of CT as a semiotic and cultural mediator carries direct implications for the structural organization of mathematics education. Moving from conceptual foundations to pedagogical practice requires deliberate shifts in curriculum design, teacher preparation, assessment paradigms, and equity frameworks. This section outlines how these dimensions must be reconfigured to sustain the =epistemological potential of CT.

5.1 Curricular reorientation

Traditional mathematics curricula, often organized around rigid disciplinary boundaries and sequential content delivery, limit opportunities for authentic problem-solving. Reorienting these structures toward inquiry-based, problem-centered learning enables mathematics to function as a dynamic system of knowledge mobilized for real- world interpretation (Chan et al., 2023; Weintrop et al., 2016). This shift does not dissolve disciplinary identities but fosters intentional bridges that position mathematical reasoning within broader epistemic networks. Learning tasks should be designed to activate semiotic mediation and collaborative inquiry, such as computational modeling of ecological systems or urban dynamics. Such integrative experiences move students beyond procedural rehearsal, engaging them in authentic intellectual work that mirrors the complexity of contemporary challenges (Finch et al., 2021; Kirk et al., 2023).

5.2 Teacher education

Educator preparation emerges as a critical lever for sustainable transformation. Many mathematics teachers have been trained within disciplinary silos and may initially perceive CT as an external technical requirement rather than a pedagogical resource (Israel et al., 2022; Kite & Park, 2023). Professional development must therefore prioritize conceptual clarity alongside pedagogical strategy, helping teachers recognize CT as a mode of reasoning that enriches mathematical inquiry. Effective preparation programs should model the integration of computational artifacts into classroom discourse, equip educators with frameworks for facilitating iterative problem-solving, and cultivate their capacity to design tasks that promote semiotic externalization and transdisciplinary connections.

5.3 Assessment practices

Conventional assessment models, which predominantly measure procedural accuracy and recall, are misaligned with the epistemic processes fostered by CT-infused mathematics education (Grover, 2017; Weintrop et al., 2021). Evaluative frameworks must be redesigned to capture the dynamics of knowledge objectification, collaborative reasoning, and transdisciplinary application. Process-oriented approaches – such as learning portfolios, project-based evaluations, and reflective documentation – offer more valid mechanisms for tracking how students construct, test, and refine computational-mathematical models. These practices shift the evaluative focus from product compliance to cognitive development, aligning assessment with the iterative and dialogic nature of CT.

5.4 Equity and access

The integration of CT into mathematics education carries significant implications for educational equity. When CT is narrowly equated with coding proficiency, access becomes contingent on prior technological exposure, inadvertently reproducing existing socioeconomic disparities (Tran, 2019). Conversely, framing CT as a form of reasoning and semiotic practice democratizes participation. By emphasizing abstraction, modeling, and collaborative problem-solving over language-specific syntax, educators can create inclusive learning environments that value diverse cognitive approaches and prior knowledge. This perspective positions CT not as a barrier but as a catalyst for broadening mathematical participation across varied student populations.

5.5 Guarding against reductionism

A persistent challenge in educational policy and practice is the conflation of CT with programming instruction (Ou et al., 2023). While coding provides a valuable context for developing computational skills, equating the two obscures the broader epistemological dimensions of CT. Reductionist interpretations strip CT of its capacity to function as a semiotic mediator, an ethos-shaping practice, and a bridge to transdisciplinary inquiry. Sustaining the transformative potential outlined in this article requires deliberate resistance to this narrowing. Curriculum designers, policymakers, and educators must prioritize CT as a cognitive and cultural framework, ensuring that its integration into mathematics education remains anchored in meaning-making, critical inquiry, and epistemic flexibility rather than technical compliance.

5.6 Broader educational aims

Collectively, these implications reposition mathematics education within the broader aims of 21st-century learning. In an era defined by interconnected complexities, students require not only disciplinary knowledge but also the adaptive competence to navigate uncertainty, synthesize diverse perspectives, and engage in sustained inquiry (Dwijayanti et al., 2026). By aligning curriculum, teacher education, assessment, and equity initiatives with the epistemological affordances of CT, mathematics education can evolve into a more integrative, responsive, and culturally situated practice.

5.7 Educational contextualization, empirical evidence and pedagogical strategies

Translating the CT construct from a theoretical framework into classroom practice requires explicit anchoring in real educational contexts, where educators routinely face challenges such as curricular fragmentation, the scarcity of validated assessment instruments, and pedagogical resistance to shifting away from strictly disciplinary, rule-based paradigms. Recent empirical studies in K–12 settings corroborate the necessity of repositioning CT beyond syntactic programming, framing it instead as an epistemic practice mediated by cultural artifacts and iterative cycles of inquiry.

In Nigerian upper-secondary education, Yusuf et al. (2026) developed and validated the *Quantum Literacy Test* (QLt) with 819 students, demonstrating that extending CT into non-binary domains (e.g., superposition, entanglement, quantum algorithms) necessitates moving beyond self-reported assessments toward robust psychometric instruments. Their findings indicate that domain-specific literacy is only partially predicted by general cognitive abilities, underscoring the need for targeted pedagogical strategies that externalize abstract concepts through semiotic mediation. Concurrently, in Israeli elementary schools, Shamir and Levin (2026) implemented a 60-hour constructionist curriculum aligned with the AI4K12 framework, engaging 209 students (ages 11–12) in building predictive machine learning systems. Their results revealed statistically significant gains in ML-related computational concepts, practices, and perspectives, demonstrating that early, scaffolded exposure to data-driven paradigms is both pedagogically viable and cognitively transformative.

These empirical experiences offer concrete strategies that align directly with the theoretical mechanisms proposed in this article:

I. Semiotic mediation and conceptual objectification**:** Both studies operationalize abstract reasoning through tangible artifacts and visual representations that function as semiotic bridges. Yusuf et al. (2026) employ unplugged activities – such as spinning doughnuts to model superposition, reversibility cards, human simulations of quantum gates, and numbered balloons for search algorithms – that render mathematical-computational logic visible, negotiable, and open to collective refinement. Similarly, Shamir and Levin (2026) introduce a *Feature Mapping* task, wherein students collaboratively construct mind maps of visual attributes prior to computational modeling. This activity explicitly externalizes the decomposition process, aligning directly with the Theory of Objectification's emphasis on making internal reasoning socially shared and materially anchored.

II. Cultivating an investigative ethos: The iterative nature of CT is materialized through cycles of training, validation, and model tuning that systematically normalize error and epistemic uncertainty. Shamir and Levin (2026) highlight the *Definitive Output* strategy, which empowers students to establish their own criteria for model adequacy. This shifts the focus from algorithmic perfection to processual understanding, thereby sustaining self-efficacy amid non-deterministic outcomes. Yusuf et al. (2026) observed that collaborative debugging and hypothesis revision during hands-on simulations fostered a classroom climate of intellectual risk-taking and mutual accountability, directly mirroring the investigative ethos proposed in this article.

III. Authentic transdisciplinary emergence: Rather than being confined to isolated computer labs, these activities emerge from complex, open-ended problems that demand the simultaneous mobilization of mathematics, scientific reasoning, and ethical reflection. The detection of algorithmic bias in training datasets (Shamir & Levin, 2026) and the modeling of search efficiency or cryptographic principles (Yusuf et al., 2026) require learners to integrate formal mathematical structures, empirical scientific principles, and critical awareness of societal impact. In doing so, transdisciplinarity ceases to be an imposed curricular mandate and instead becomes an epistemic necessity of authentic computational practice.

The integration of these empirical findings and pedagogical strategies demonstrates that CT, when operationalized through intentional semiotic mediation, collaborative debugging cycles, and anchoring in authentic problems, fully realizes its potential as an epistemological catalyst. To advance the empirical validation of the theoretical framework proposed herein, future research should prioritize longitudinal and design-based methodologies that monitor not only technical proficiency, but also the evolution of conceptual objectification, the transformation of classroom ethos, and students' growing capacity to navigate knowledge domains with epistemic flexibility.

6. CONCLUSION

This article has advanced a theoretical integration positioning computational thinking as a transformative mediator in mathematics education. By anchoring CT within a cultural-historical semiotic framework, the analysis demonstrates how computational practices reconfigure mathematical learning from procedural reproduction to dynamic, culturally situated meaning-making. Rather than functioning as a technical supplement, CT operates through three interconnected pathways: it expands the semiotic resources available for externalizing reasoning, fosters classroom norms that treat iteration and error as constitutive of inquiry, and renders transdisciplinary engagement an organic outcome of authentic problem-solving. Together, these mechanisms illustrate how computational practices can bridge formal mathematical structures and the complex, interconnected challenges of contemporary contexts.

Realizing this potential requires deliberate structural alignment. Curricular designs must prioritize problem-centered inquiry, teacher preparation should foreground CT as a mode of reasoning rather than syntax mastery, and assessment frameworks need to capture epistemic processes alongside procedural competence. Equally critical is maintaining a conception of CT that resists reduction to programming, ensuring its integration remains aligned with broader educational aims of criticality, collaboration, and epistemic flexibility.

As a theoretical essay, this study does not address the empirical dimensions of classroom implementation, which constitutes a methodological limitation. Nevertheless, the theoretical articulation developed here establishes a foundation that now requires systematic empirical validation. To advance the field beyond theoretical articulation and performance-based metrics, future research should prioritize four interconnected methodological pathways:

I. Longitudinal investigations of epistemic development: Sustained tracking of classrooms integrating CT into mathematics is needed to examine how students' semiotic reasoning, collaborative dispositions, and transdisciplinary problem-solving capacities evolve over time. Longitudinal designs would help clarify causal pathways between CT-infused pedagogies and the gradual cultivation of na investigative ethos, moving beyond crosssectional outcome measures to capture deeper cognitive and cultural shifts in how learners engage with mathematical knowledge.

II. Cross-cultural and cross-level comparative studies: The mediating role of CT may be moderated by contextual variables, including institutional norms, cultural attitudes toward error and collaboration, and varying levels of technological access. Comparative research across educational stages and diverse sociocultural settings would help delineate which mechanisms of semiotic mediation and ethos transformation are robust and which require contextual adaptation. Such studies would refine the framework's applicability and inform culturally responsive implementation strategies.

III. Intersections with emerging technologies: as artificial intelligence, data science, and advanced simulation environments become increasingly accessible, research must explore how these tools expand the semiotic repertoire available for mathematical objectification. Empirical studies should investigate how AI-assisted modeling, automated feedback systems, and interactive data visualization reshape students' engagement with abstraction, algorithmic design, and cross-domain reasoning. This line of inquiry would clarify whether emerging technologies amplify or complicate the epistemological functions of CT.

IV. Design-based research for instructional sequences: Bridging theory and practice requires iterative, classroom-embedded research. Design- based studies can develop, test, and refine instructional sequences that operationalize the principles of semiotic mediation, iterative inquiry, and transdisciplinary integration outlined in this article. Such research would yield actionable pedagogical models, assessment rubrics, and teacher facilitation strategies that translate theoretical constructs into scalable classroom practices. Mixed-methods approaches, combining multimodal learning analytics with ethnographic observation, would be particularly well-suited to capture both quantitative learning progressions and the qualitative nuances of collaborative meaning-making.

Advancing this empirical agenda will not only test the theoretical propositions advanced here but also generate evidence-based guidelines for curriculum design, teacher preparation, and assessment reform. By grounding the conceptual framework in systematic classroom research, the field can move toward a more cohesive, culturally responsive, and epistemologically integration of CT in mathematics education.

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